

Exercise 10

In Exercises 7–10, write a formula for $f \circ g \circ h$.

$$f(x) = \frac{x+2}{3-x}, \quad g(x) = \frac{x^2}{x^2+1}, \quad h(x) = \sqrt{2-x}$$

Solution

Evaluate the function composition.

$$\begin{aligned} f \circ g \circ h &= f(g(h(x))) \\ &= f(g(\sqrt{2-x})) \\ &= f\left(\frac{(\sqrt{2-x})^2}{(\sqrt{2-x})^2+1}\right) \\ &= f\left(\frac{(2-x)}{(2-x)+1}\right) \\ &= f\left(\frac{2-x}{3-x}\right) \\ &= \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} \\ &= \frac{\frac{2-x+2(3-x)}{3-x}}{\frac{3(3-x)-(2-x)}{3-x}} \\ &= \frac{2-x+2(3-x)}{3-x} \times \frac{3-x}{3(3-x)-(2-x)} \\ &= \frac{2-x+2(3-x)}{3(3-x)-(2-x)} \\ &= \frac{8-3x}{7-2x} \end{aligned}$$